1) Show using Maxwell's Eqsns and the definitions from Chilwell's paper of $u, v, w$ that

$$v = \frac{\chi}{ik\alpha} \frac{d\chi}{dx}, \quad w = \beta y u, \quad u = \frac{1}{ik\gamma\alpha} \frac{dv}{dx}$$

for both TE and TM polarization.

b) Using the boundary conditions we already know for $E$ and $B$, solve for boundary conditions at an interface for $u, v, w$. Once again, do this for both polarizations.

c) For a right going wave (positive $k_x$), show

$$u^+ = \gamma u^+ \quad \text{(or)} \quad \left( \begin{array}{c} u^+ \\ v^+ \end{array} \right) = \left( \begin{array}{c} 1 \\ \gamma \end{array} \right) u^+$$

Show for the left going wave

$$\left( \begin{array}{c} u^- \\ v^- \end{array} \right) = \left( \begin{array}{c} 1 \\ -\gamma \end{array} \right) u^-$$

d) Use the fact that

1. $u$ can be written as $u = u^+ + u^-$ (or) $\left( \begin{array}{c} u \\ v \end{array} \right) = \left( \begin{array}{c} 1 \\ \gamma \end{array} \right) u^+ + \left( \begin{array}{c} 1 \\ -\gamma \end{array} \right) u^-$

2. $u^+(x_{j-1}) = u^+(x_j) e^{-i k\alpha_j (x_j - x_{j-1})}$

   $$u^-(x_{j-1}) = u^-(x_j) e^{+i k\alpha_j (x_j - x_{j-1})}$$

   Explain why these are true.

   Define $\Phi_j = k\alpha_j (x_j - x_{j-1})$

3. $e^{\pm i k\alpha_j (x_j - x_{j-1})} = \cos(\Phi_j) \pm i \sin(\Phi_j)$

   to show that $\left( \begin{array}{c} u_{j-1} \\ v_{j-1} \end{array} \right) = M_j \left( \begin{array}{c} u_j \\ v_j \end{array} \right)$ where

   $$M_j = \left( \begin{array}{cc} \cos \Phi_j & -\frac{i}{\gamma_j} \sin \Phi_j \\ -i \gamma_j \sin \Phi_j & \cos \Phi_j \end{array} \right)$$
2) a) Write down $r_{cs}$ and $t_{cs}$ from the paper.
   b) Write down $R$ and $T$ from the paper and explain what they are.
   c) Write down the time averaged power flows from the paper.
   d) Write down $\Phi_{cs}$ from the paper, and explain its physical significance.
   e) Show that if there is only a simple interface (no layering, which means $M = I = (1,1)$) that $T$ and $R$ reduce to what we solve for in class. Show it works for both $TE$ and $TM$ waves.

3) Using the Mathematica code, create a $\frac{1}{4}$ wave stack consisting of a glass ($n=1.5$) cover and substrate, and SiN layers ($n=1.8$) inside. Make the $\frac{1}{4}$ wave stack optimal for normal incidence and yellow light ($\lambda_0 = 580$ nm). Remember $\lambda_{glass} = \frac{\lambda_0}{n}$, etc.
   a) Calculate $R, T$ for $m=2, 10, 100$, where $m$ is the number of repeats of $\underset{SiN}{\text{Glass}}$.
   b) Find $m$ where $R$ is first above 90%.
   c) For the $m$ in part (b), create a graph of $R, T$ (on the same axis) as you vary the wavelength (without changing thicknesses) from 300 nm to 800 nm (the whole visible spectrum). Is this a good reflector for the whole range (could it be a mirror?)?
   d) For the $m$ in part (b), create a graph of $R, T$ varying the incident angle ($\lambda_0 = 580$ nm again) from $0^\circ$-$90^\circ$. Does this mirror work well off-axis?
(a) Using the \( \frac{1}{4} \) wave stack from (3), find \( m \) for 99% reflection.

(b) Now create two of those stacks on top of each other. Basically make \( m=2m \). What is \( R \) now?

(c) Do the same as in (b), but this time make a space between the first \( m \) stacks and the second \( m \) stacks that is not \( \frac{1}{4} \) wavelength, but \( 1 \) wavelength.

What is \( R \) and \( T \) now? What you have just made is a resonant cavity. We'll learn more about that later.

5) a) Say you have a glass substrate and you want to eliminate reflection at normal incidence with a single layer on the glass. So your structure looks like:

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air
layer(n_2)
glass (n=1.5)
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Analytically solve for \( M \) in terms of constants \( n_2 \) and \( n_2 \). Then set \( \rho_2 = 0 \) to find a relationship for \( n_2 \) and \( m_2 \).

Use normal incidence. This is an anti-reflection coating.

b) Make a graph for one case with \( n_2 \) varying \( \theta_2 (0-90^\circ) \).