3. A cylindrical capacitor of capacitance C, and inner and outer radii a and b, is charged to voltage V. Using Gauss's law derive the electric field in the region between a and b in terms of C and V. Assume no fringing fields.

\[ \varepsilon E \cdot d\mathbf{a} = \frac{dQ}{\varepsilon_0} \]

\[ \varepsilon \mathbf{E} \cdot d\mathbf{a} = 0 \text{ on both end caps}, \quad \varepsilon \mathbf{E} \cdot d\mathbf{a} \text{ for all sides on body of cylinder} \]

\[ \int \mathbf{E} \cdot d\mathbf{a} = \varepsilon \int \mathbf{E} \cdot d\mathbf{a} \]

\[ E_s d\mathbf{a} = E \cdot 2\pi L = \frac{Q}{\varepsilon_0} \]

\[ E = \frac{Q}{2\pi \varepsilon_0 L} \quad \text{but} \quad C = \frac{Q}{V} \]

So \( Q = CV \)

Three coupled ODE's with boundary conditions \( r_0, r_0, q, v, x, y \)

4. The charge configuration shown creates an electrostatic lens for an electron microscope. How would you determine the trajectory of an electron with initial velocity \( \mathbf{v}_0 \) and position \( r_0 \)?

**Newton's law**

\[ \mathbf{F} = q \mathbf{E}(x, y, z) = m \frac{d\mathbf{v}(x, y, z)}{dt} \]

\[ \mathbf{v}(x, y, z) = \mathbf{v}_0(t) \]

**Newton's law is 3 eqns**

\[ q E_x(x, y, z) = m \frac{dx(t)}{dt} \]

\[ q E_y(x, y, z) = m \frac{dy(t)}{dt} \]

\[ q E_z(x, y, z) = m \frac{dz(t)}{dt} \]

3 1st order ODE's

Solve using initial velocity and do again to get trajectory. This reduces to \( 3 \text{ 2nd order ODE's} \)

Compare with the example \( \mathbf{F} = -k \mathbf{x} = m \frac{d^2\mathbf{x}}{dt^2} \)

\( x = A \cos(\sqrt{2} t + \phi) \) where \( A \neq 0 \) are determined by initial conditions.

OR solve numerically as discussed in class (see lecture notes).

**SOLN is not** \( \mathbf{v}(t) = \mathbf{v}_0 + \mathbf{a} t - \frac{1}{2} \mathbf{a} t^2 \)