2. (12 points) The potential at the surface of a sphere is $V(\theta) = V_0 \cos \theta$. Determine an integral expression for the voltage outside the sphere. The general radial solution is $R(r) = A_l r^l + \frac{B_l}{r^{l+1}}$, while the general angular solution is $P_l(\cos \theta)$ where $l$ is an integer. Note: $\int_{-1}^{1} P_l(x) P_m(x) dx = \frac{2}{2l+1} \delta_{lm}$.

Separation of variables solution doesn't satisfy boundary condition. Use superposition principle and orthogonality of $P_l(\cos \theta)$ to construct a general solution. Outside the sphere:

$A_l r^l \to 0$ as $r \to \infty$ so $A_l = 0$.

$V(r, \theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos \theta)$ at boundary $V_0 \cos \theta = \sum \frac{B_l}{R^{l+1}} P_l(\cos \theta)$.

Multiply both sides by $P_m(\cos \theta)$ and integrate to use orthogonality:

$\int_{-1}^{1} V_0 P_m(x) P(x) dx = \sum \frac{B_l}{R^{l+1}} \int_{-1}^{1} P_m(x) P_l(x) dx = \frac{B_m}{R^{m+1}} \frac{2}{2m+1}$

$B_m = \frac{R^{m+1}}{2} \int_{-1}^{1} V_0 P_m(x) dx$.